Temperature Measurements in a Spherical Field: Transfer Coefficients and Corrections for Thermocouples in Boundary Flows

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A study was made of the deviation between the temperature of thermocouple junctions and the temperature of the fluid surrounding them which arises as a result of conduction along the thermocouple leads. Measurements around a ½-in. heated sphere in the plane of the equator normal to an air stream having a velocity of 16 ft./sec. were made with a 0.001-in.-diam. thermocouple of platinum and platinum rhodium. Air temperatures in the boundary flows of the sphere were predicted from the indications of the thermocouple. The method of prediction employed also determined the heat transfer coefficients for the thermocouple wire in the boundary layer. These local heat transfer coefficients are lower than those observed in a uniform stream having a velocity equal to the estimated local velocity in the boundary layer.

Measurement of the local temperature of a fluid in the presence of large temperature gradients is a major experimental problem encountered in many heat transfer studies. The differential equations used to describe the temperature in and around temperature-sensing devices are usually simple, but the analytical solution to these equations is often impossible because of unusual geometry or irregular boundary conditions. Carslaw (*4*) and Ingersoll (*13*) presented equations and a few solutions applicable to heat-conduction problems encountered when thermocouples are used.

Several investigators have studied the problem of conduction in thermocouples. Johnson (*15*) determined analytically the thermocouple temperature for each of five air-temperature distributions along the thermocouple leads. Boelter (*3*) corrected the readings of thermocouples mounted on flat plates in heaters. Hsu (*10*) presented an analogue method for correcting thermocouple readings taken in the boundary layer surrounding a heated sphere. Extensive studies of corrections for hot-wire anemometers were made by King (*17*), whose results in many cases can be applied to thermocouples because of similar geometry and boundary conditions. The application of all these results presumes a knowledge of the heat transfer coefficient for the wire. The present paper describes a method which does not require an independent knowledge of the heat transfer coefficient for predicting point air temperatures from the indications of small thermocouples.

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METHODS

A study of the effect of free-stream turbulence on the local thermal and material transfer from spheres has been under way for a number of years. To determine the local thermal flux from the surface of a silver sphere, temperatures were measured as a function of position in the fluid adjacent to the surface. Temperature traverses were obtained by moving a thermocouple junction through the thermal-boundary layer to the surface of the sphere. Traverse data of this type were presented by Hsu (*10, 12*) and Short (*21*). Junction temperatures from a typical experimental traverse are shown in Figure 1. The term *junction temperature* refers to the measured temperature as distinguished from the true *air temperature* at the same point in space. The term *wire temperature* denotes temperatures in the thermocouple wire at points other than the junction. The *thermocouple correction* is the difference between the air temperature and the junction temperature. The term *surface temperature* represents the temperature determined with thermocouples mounted within the surface of the sphere.

The difference between the junction temperature and the surface temperature, as shown in Figure 1, indicates that the junction temperatures are considerably lower than the corresponding air temperatures. Earlier investigators believed that thermal conduction along the thermocouple wires caused these deviations.

EQUIPMENT

The probe thermocouple used for this investigation was made of 0.001-in.-diameter wires of platinum and of platinum–10% rhodium. The thermocouple was soldered to the tips of two needles on a probe, as shown in Figure 2. Wires of this size were chosen because they were the smallest that could be butt-welded in an oxy-natural gas flame to form a smooth cylindrical junction. No irregularities at the junction were observed under a 36-power binocular microscope. Thermocouples which were made from wires 0.0003 in. in diameter showed nonuniform junctions.

The probe thermocouple could be moved about the sphere parallel to each of the three coordinate axes shown in Figure 3. Dial indicators reading to 0.001 in. measured the displacement of the probe in the two directions normal to flow. The thermocouple wires were always parallel to the y axis, which is normal to the air stream.

The 0.5-in. sphere used in this investigation was employed by Baer (*1*), Hsu (*12*), and Sato (*18*) in earlier studies of heat transfer from spheres. The surface of the sphere was formed by two hemispheres of polished silver 0.016 in. thick fitted over a copper core on which an electrical

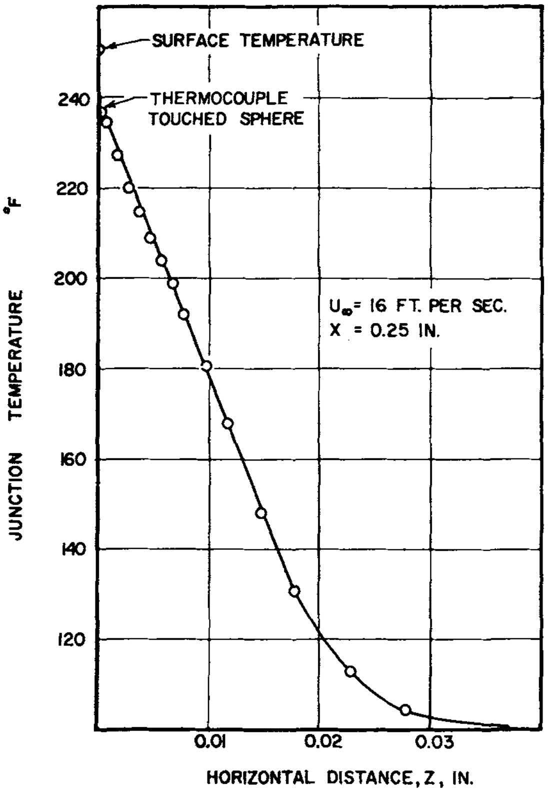

Fig. 1. Typical junction-temperature traverse.

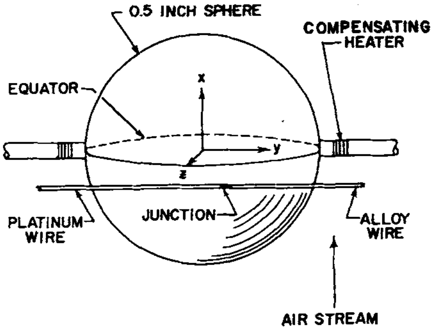

Fig. 3. Orientation of probe thermocouple and sphere.

heater was wound. Four copper constantan thermocouples were embedded in the inner surface of the silver shell to measure surface temperatures. The sphere was mounted on stainless steel tubes 0.096 in. in diameter provided with compensating heaters. When the heaters were properly adjusted, no measurable amount of energy was conducted to or from the sphere along the supporting tubes.

The air stream in which the sphere was suspended was supplied by equipment described by Corcoran (*6*) and Hsu (*11*). The air stream was maintained at a velocity of 16 ft./sec., a temperature of 100°F., and an apparent level of turbulence of 0.013. The level of turbulence is defined as the ratio of the root-mean-square of the lateral fluctuating velocity component to the average longitudinal velocity. The turbulence value was established by Sato (*18*) using the data of Schubauer (*19*) in conjunction with measurements of the divergence of the temperature profile in the wake of a heated wire.

The copper-constantan and platinum–platinum-rhodium probe thermocouples were calibrated in the laboratory against a standard resistance thermometer. These calibrations, which indicated a maximum deviation of 0.8°F. from reference values (*20*), were used in obtaining all temperatures. The reference junction for all thermocouples was at the ice point.

ANALYSIS

The analysis of thermocouple corrections which follows is presented in three parts. First, the assumptions pertinent to the analysis are discussed; then a differential equation to describe temperature in the vicinity of the thermocouple, with only conductive and convective thermal transport taken into account, is developed and discussed; and finally a method of solving the differential equation to obtain air temperatures from junction temperatures is presented.

Assumptions

Thermal transport by radiation from both the sphere and the thermocouple is small compared with the transport by convection. At 212°F. the total emissivity of polished silver is 0.02 and that of unoxidized platinum is 0.047 (*9*). Since the wire surface near the junction became oxidized during welding, it is believed that the emissivity was higher than 0.047. Radiation transfer from an unoxidized platinum wire at 238.8°F., the maximum junction temperature observed experimentally, to surroundings at 80°F. is 0.0035 B.t.u./(sq. ft.) (sec.). The minimum heat transfer coefficient

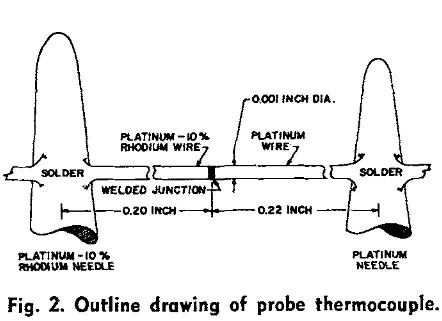

Fig. 2. Outline drawing of probe thermocouple.

encountered in this investigation was 0.04 B.t.u./(°F.) (sq. ft.) (sec.). To transfer energy at a rate of 0.0035 B.t.u./(sq. ft.) (sec.) from a body having this heat transfer coefficient, a temperature difference of 0.09°F. is required. Under any of the experimental conditions encountered, the thermocouple correction due to radiation effects is of this magnitude.

Additional errors are expected to arise from temperature jump (*7, 16, 22*) and stagnation effects. The maximum temperature jump for a surface gradient of 6,000°F./in. was calculated from the empirical equation of Kennard (*16*) to be 0.041°F. A maximum stagnation temperature rise at a gross velocity of 16 ft./sec. was predicted from potential flow theory to be 0.048°F.

The difference in temperature between the inner and outer surfaces of the silver shell of the sphere was computed to be 0.008°F. at the maximum thermal flux. The internal thermocouples were embedded in a nearly isothermal surface; therefore errors due to conduction along these thermocouples are negligible.

Since the position of the thermocouple junction is known to within ±0.0005 in. in the z direction, an uncertainty of 0.3°F. in the junction temperature exists at the maximum gradient observed. It is thus seen that the effects discussed above are of the same order of magnitude as this uncertainty and may be neglected. Hence the difference between the surface temperature and the temperature of the junction when nearly in contact with the surface, which was found to be 12.28°F., is considered equal to the thermocouple correction at the surface.

Differential Equation

To determine the thermocouple correction at a point in the air stream, a differential equation describing the temperature in the vicinity of the wire was developed. The convective thermal transfer rate per unit area results from the difference between the local air

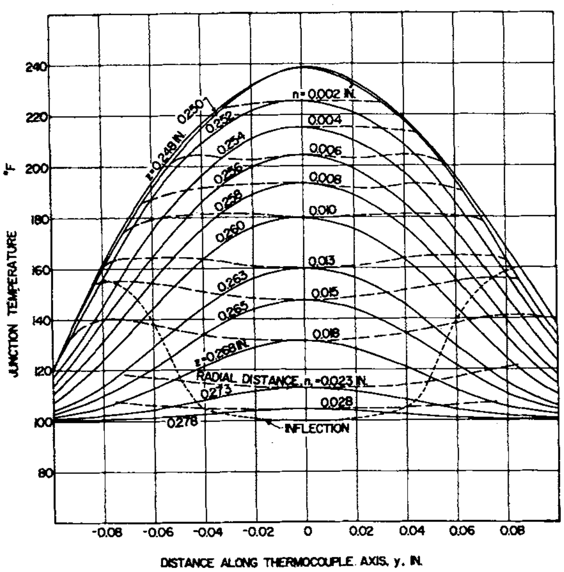

Fig. 4. Experimental junction temperatures in plane of equator.

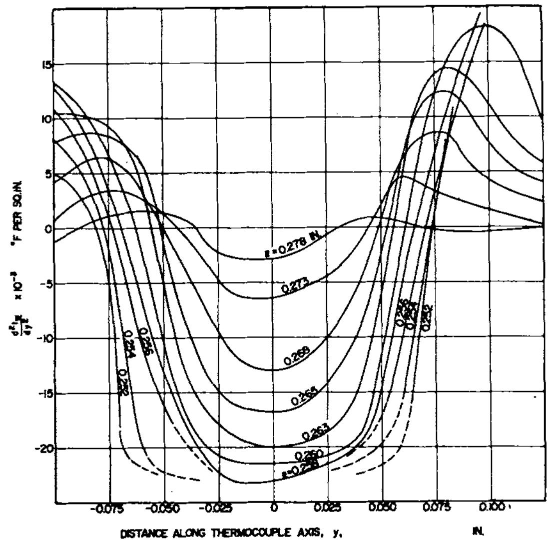

Fig. 5. Second derivative of experimental junction temperatures.

temperature and the wire temperature as indicated by Equation (1):

$$\mathring{Q} = h(t_a - t_w) \qquad (1)$$

The heat transfer coefficient of Equation (1) is based on the difference between air temperature and wire temperature rather than on the difference between free-stream temperature and wire temperature, as is usually the case. This distinction is of consequence when high-temperature gradients surround the wire.

The convective thermal transfer rate per unit length of wire is

$$\mathring{Q} = \pi h D(t_a - t_w) \qquad (2)$$

For steady state to exist, this must be equal to the rate of energy loss per unit length of wire by conduction along the wire, which is given by

$$\mathring{Q} = -\frac{\pi D^2}{4}\frac{d}{dy}\left(k\frac{dt_w}{dy}\right) \qquad (3)$$

Hence combining Equations (2) and (3) and simplifying results in the following equation

$$t_a - t_w = -\frac{D}{4h}\frac{d}{dy}\left(k\frac{dt_w}{dy}\right) \qquad (4)$$

The thermal conductivity of platinum is more than twice that of the platinum-rhodium alloy (*14*). The evaluation of the temperature gradient, dt_w/dy, near the junction, and of the thermal flux through the junction, $k(dt_w/dy)$, offers some difficulty, as these factors require a knowledge of the wire temperature in the neighborhood of the junction. The thermocouple junction was moved a short distance along the axis of the wire on either side of the point to which Equation (4) was being applied to establish a temperature distribution. Movement of the junction changes one boundary condition defining the solution to Equation (4). This boundary condition is the thermal conductivity of the wire as a function of y. Each junction temperature measured in this manner is one temperature from a different solution to the differential equation. The thermal flux $k(dt_w/dy)$ established from these temperatures may be in error.

From the foregoing discussion it is apparent that the direct solution of Equation (4) without further simplifying assumptions would require a numerical method involving detailed information concerning the temperature gradients near the junction. Some further simplifications appear desirable if the method is to be useful.

Solution to the Differential Equation

To gain insight into this problem, it is assumed that the thermal conductivities of the metals in the thermocouple are constant and equal. Under these circumstances it is possible to employ an apparent, consistent thermal conductivity which will satisfy the equation at several points in the temperature field and thus afford a good approximation to the actual behavior involving two different and variable thermal conductivities. Equation (4) then becomes

$$t_a - t_w = -\frac{Dk^*}{4h}\left(\frac{d^2t_w}{dy^2}\right) \qquad (5)$$

It should be recognized that the apparent thermal conductivity which is determined from experimental measurements with thermocouples at several points in the temperature field is not any particular kind of average of the thermal conductivities of the two metals. In the case of Equation (5) the location of the junction is not one of the boundary conditions. The second derivative of wire temperature and of the thermal flux in the wire in this instance can be approximated from a knowledge of junction temperatures as a function of position. The assumption of constant thermal conductivity throughout the wire permits the evaluation of all factors on the right side of Equation (5) except the heat transfer coefficient.

The data of Cole (*5*) for values of the heat transfer coefficient as a function of velocity can be used in conjunction with the velocity-distribution equation developed by Frössling (*8*) to obtain heat transfer coefficients as a function of position. However the presence of the wire in the boundary layer introduces uncertainties in the application of the heat transfer coefficients obtained in this way. To circumvent this problem it was assumed that the velocity and temperature fields are symmetrical about the x axis. By appropriate assumptions of symmetry with respect to temperature, velocity, and hence heat transfer coefficient, it is possible to establish the temperature correction from data at the two points of symmetry. The two points through which the thermocouple may pass are designated 1 and 2. These points are such than $n_1 \neq n_2 > r$.

The resulting equation is

$$t_{a1} = \frac{t_{w1} - t_{w2}\left[\left(\frac{d^2t_w}{dy^2}\right)_1 \Big/ \left(\frac{d^2t_w}{dy^2}\right)_2\right]}{1 - \left[\left(\frac{d^2t_w}{dy^2}\right)_1 \Big/ \left(\frac{d^2t_w}{dy^2}\right)_2\right]} \qquad (6)$$

Although Equation (6) does not contain the apparent thermal conductivity, it is implicit in its derivation that a single value of the apparent thermal conductivity may be applied. Air temperatures can be computed from junction temperatures by means of Equation

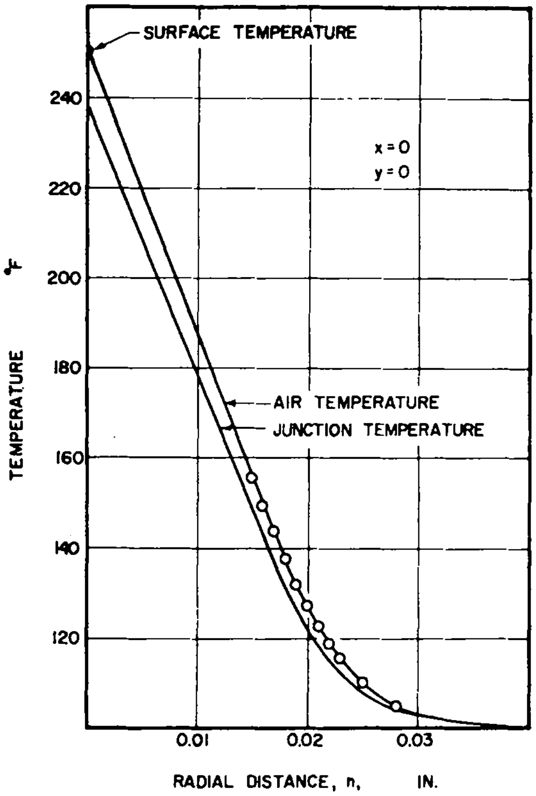

Fig. 6. Radial temperature distribution at equator.

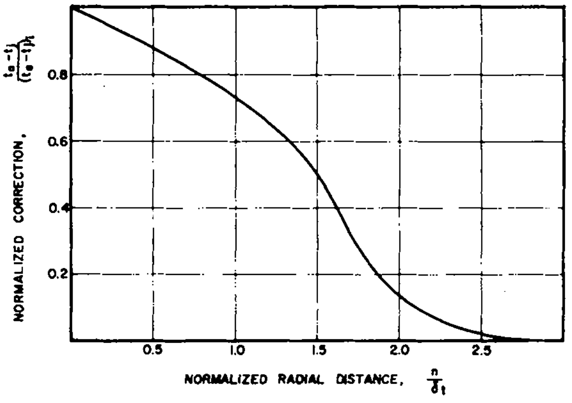

Fig. 8. Dimensionless thermocouple corrections for a 0.001-in.-diameter wire.

(6), which is subject to the following restrictions:

$$h = \phi(n) \qquad t_{w1} \neq t_{w2}$$

$$k = \text{constant} \quad \left(\frac{d^2 t_w}{dy^2}\right)_1 \neq \left(\frac{d^2 t_w}{dy^2}\right)_2$$

As a result of the existence of a number of assumptions, it appears advantageous to utilize Equation (6) in regions where the corrections are small. In the present instance Equation (5) was employed to delineate the locus of states where the correction was equal to zero, which corresponded to $d^2t_w/dy^2 = 0$.

EXPERIMENTAL RESULTS

Figure 4 illustrates the data obtained from experimental traverses. The temperatures, which were nearly linear with respect to z over a large portion of the field, were smoothed graphically. This operation yielded a standard deviation of approximately 0.15°F. The second derivatives of junction temperature with respect to y were computed from the smoothed data by the use of a three-point approximation (23). The curves for the second derivative in this region are shown in Figure 5. A five-point approximation (23) was tried, but it was found that use of fewer points gave more consistent results, because of the limited number of total points available. Undoubtedly the five-point approximation gave more accurate results near the center of the curve but apparently gave biased values at each end.

Since $d^2t_w/dy^2 = 0$ at an inflection point, it follows from Equation (5) that the thermocouple correction should be zero at such a point. By a numerical solution of Equation (5) it was established that the difference between the air temperature and junction temperature at an inflection point was less than 0.1% of the difference between the free stream and the surface temperatures if a shift in position of 0.0013 in the y direction was made. By the use of this corrected coordinate system, the junction temperature was assumed equal to the air temperature where $d^2t_w/dy^2 = 0$.

The line of inflection shown in Figure 4 was obtained from Figure 5. The intersection of each constant-radius curve with the inflection curve corres-

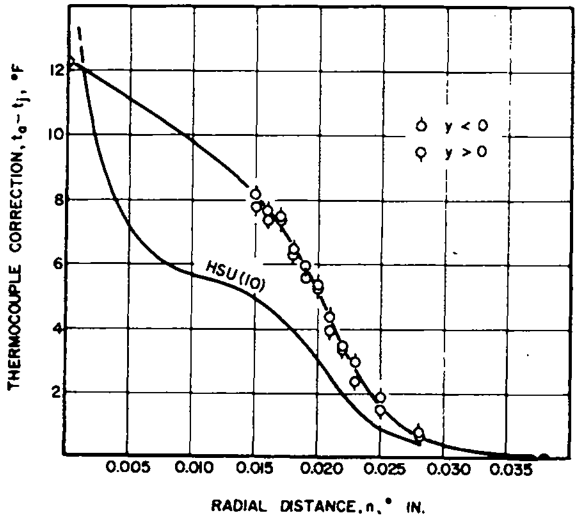

Fig. 7. Thermocouple corrections for a 0.001-in.-diameter thermocouple.

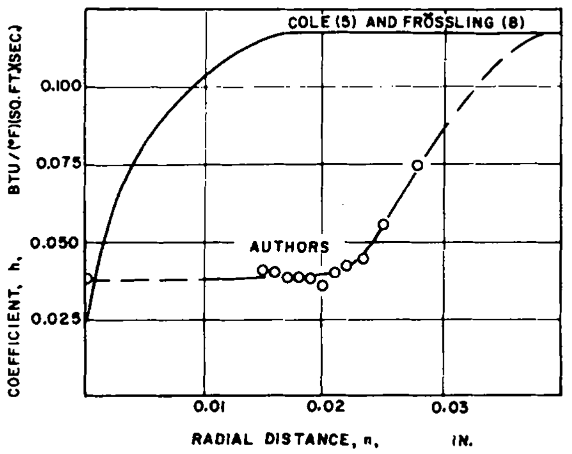

Fig. 9. Experimental convective heat transfer coefficients for a 0.001-in.-diameter wire near a 0.5-in. sphere.

ponds to the air temperature at that radius. The arithmetic average of the two air temperatures is shown in Figure 6. In Figure 7 the thermocouple corrections are compared with the thermocouple corrections computed by the method of Hsu (*10*) for the same conditions of flow. As the inflection curve of Figure 4 did not approach the sphere closer than 0.014 in. the air temperature, and consequently the thermocouple correction, could not be established in the region closer to the sphere than 0.014 in. The corrections calculated in the manner described are in good agreement with the value determined from the junction temperature near the interface and the surface temperature.

The measured junction-temperature gradient normal to the sphere at the equator is 5,730°F./in. This value is the slope of the junction-temperature curve in Figure 5 in the range $0 < n < 0.012$. The air-temperature gradient over the same range is 6,010°F./in. The local thermal flux established from the junction-temperature gradient is 4.9% low in this instance. The error in the local thermal flux determined in this manner varies with position around the sphere because, as observed in other heat transfer studies (*12*), the thermocouple correction at the surface decreases with increasing polar angle measured from the stagnation point.

Figure 8 shows values of thermocouple corrections which were obtained by normalizing the thermocouple corrections in Figure 6 with respect to the correction at the surface. The radial distances were normalized with respect to the thermal boundary-layer thickness, defined as

$$\delta_t = \int_0^\infty \frac{t_a - t_\infty}{t_i - t_\infty}\, dn \qquad (7)$$

For a fluid with constant physical properties the normalized correction may be a function of one or more of the following parameters: free-stream velocity, free-stream turbulence, diameter of sphere, and polar angle. Hsu (*12*) related temperatures in the boundary layer of a sphere to the Blasius function (*2*).

To calculate heat transfer coefficients it was necessary to obtain a suitable value of the thermal conductivity of the thermocouple. An apparent conductivity of 6.478×10^{-3} B.t.u./(sec.)(sq.ft.)(°F./ft.) was found by using a numerical solution of Equation (4) in conjunction with experimental data and assumed heat transfer coefficients of appropriate orders of magnitude.

Heat transfer coefficients for the thermocouple wire, computed by the use of the apparent thermal conductivity of platinum and platinum rhodium, are presented in Figure 9. Also shown, for comparison, are the heat transfer coefficients obtained with the equations of Frössling (*8*) and data of Cole (*5*). The large difference between the two sets of heat transfer coefficients is not surprising. As remarked earlier, the heat transfer coefficients used in this paper are based on air temperatures rather than free-stream temperatures. Close to the sphere the difference between the temperature potentials may well explain at least 75% of the discrepancy between the two sets of data. Furthermore the velocity gradient produces pressure gradients in the boundary layer of the wire, and flow parallel to the wire axis develops. Since temperature and velocity gradients at the equator are in opposite directions, this leads to a heat transfer coefficient lower than that determined in a free stream. This phenomenon may also explain the flat appearance of the curves of Figure 5 in the region near the surface of the sphere.

It is expected that the heat transfer coefficients obtained are applicable only in the vicinity of the equator, since gradient effects in other regions of the sphere are different.

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NOTATIONS

d = differential operator
D = diameter of thermocouple wire, in.
h = thermal transfer coefficient, B.t.u./(°F.)(sq.ft.)(sec.)
k = thermal conductivity, B.t.u./(°F./ft.)(sq.ft.)(sec.)
k^* = apparent thermal conductivity B.t.u./(°F./ft.)(sq.ft.)(sec.)
n = radial distance from surface of sphere, in.
$\overset{\circ}{Q}$ = thermal flux, B.t.u./(sec.)(sq.ft.)
$\underline{\overset{\circ}{Q}}$ = thermal transfer rate per unit length, B.t.u./(sec.)(sq.ft.)
r = radius of sphere, 0.250 in.
t_a = air temperature, °F.
t_i = surface temperature, °F.
t_j = junction temperature, °F.
t_w = wire temperature, °F.
t_∞ = free-stream temperature, °F.
U_∞ = free-stream velocity, ft./sec.
x,y,z = coordinate axes, in.
δ_t = thermal boundary-layer thickness, in.
$\phi(\ )$ = function of

Subscripts

i = surface of sphere
1,2 = points in the air stream

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